Midwest Home Price Index v.s. National Average from 1991-2016

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Applied Time Series Analysis, Spring 2017

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- Complex methodology used to get estimates is described by the FHFA
- Interested in comparing Midwest HPI with the national average

Original series

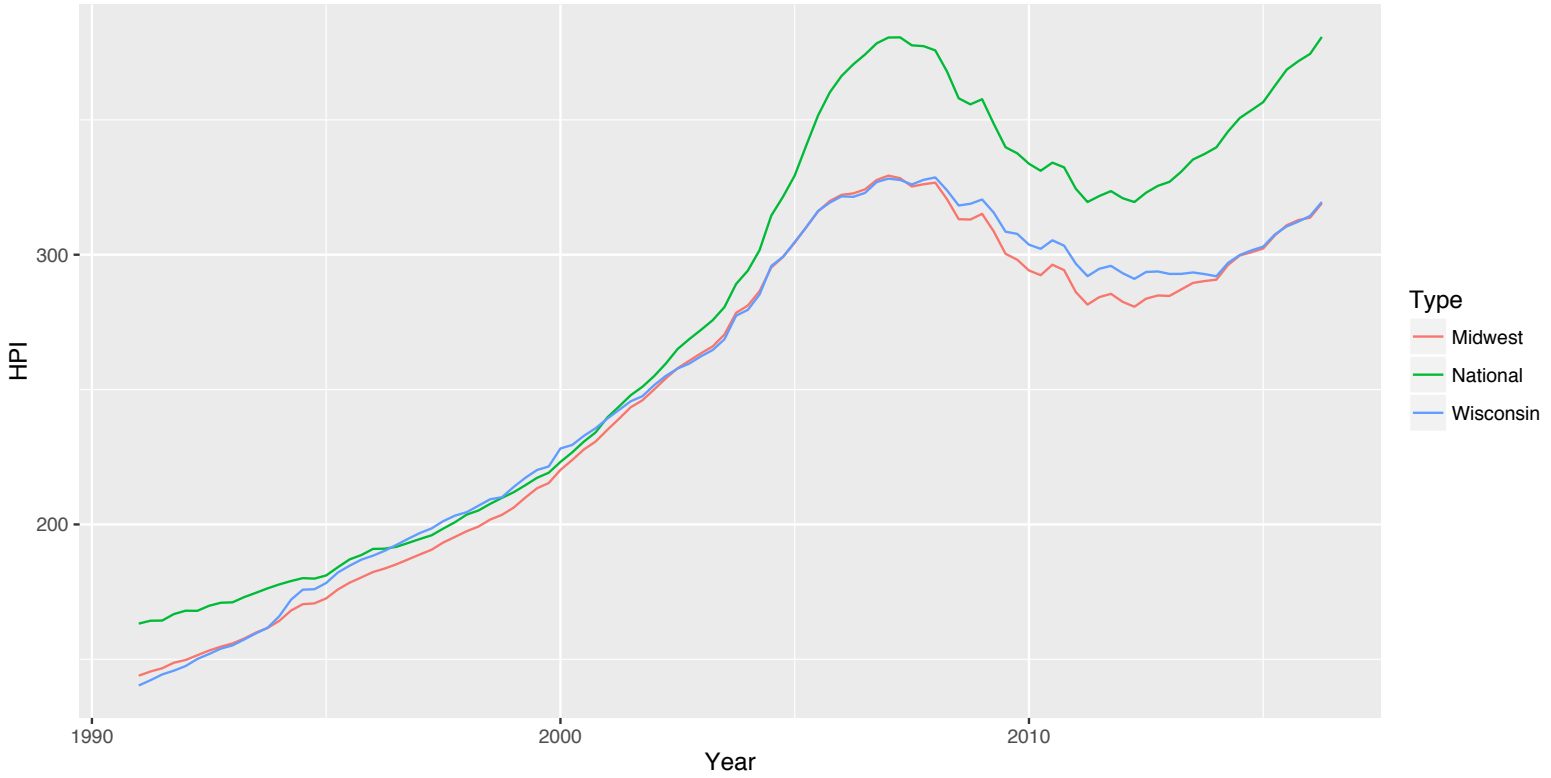


Figure: Original quarterly HPI for each of the three regions

Box Cox Transformation

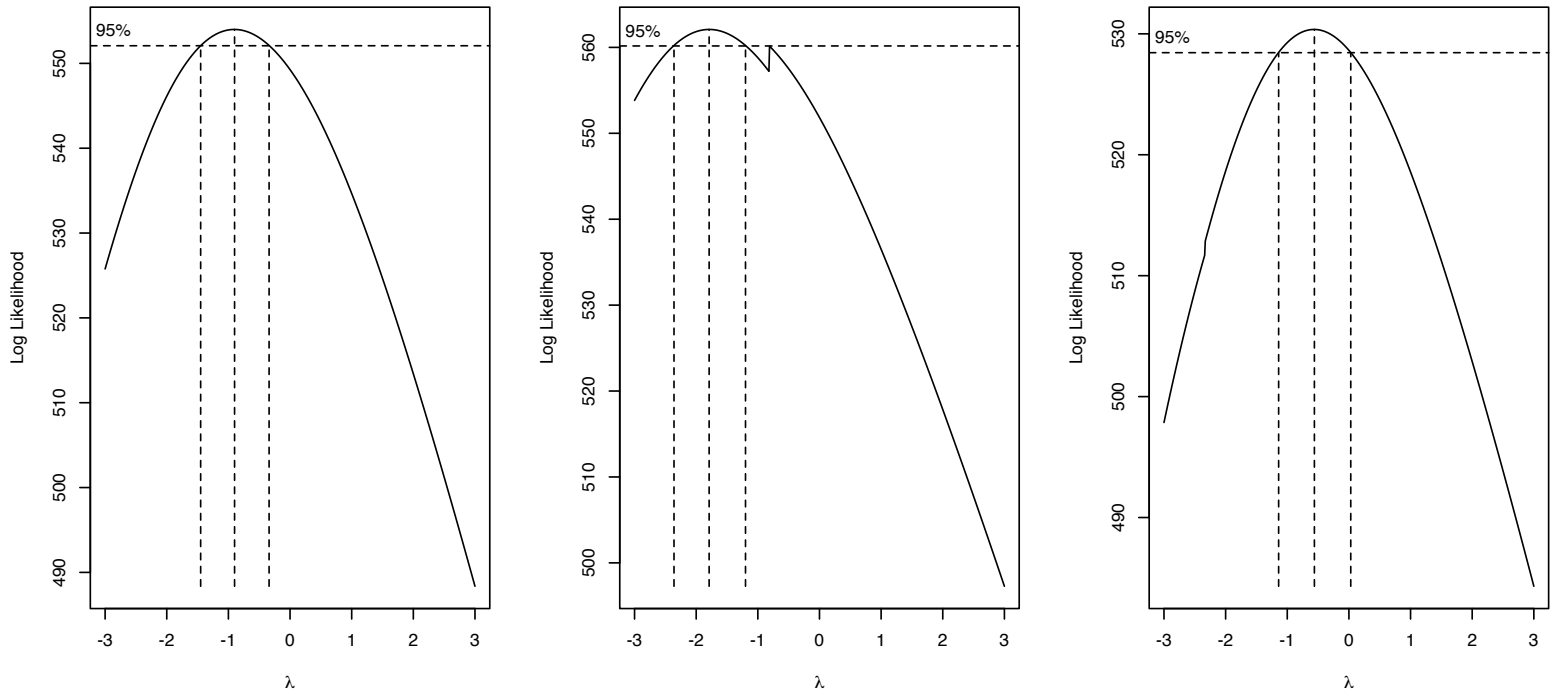


Figure: From left to right, the likelihood for a Box-Cox variance stabilizing parameter are shown for the national, Midwest, and Wisconsin series', respectively.

Box Cox Transformation

$$y^* = \begin{cases} \frac{y^\lambda - 1}{\lambda} & \text{if } \lambda \neq 0 \\ log(y) & \text{if } \lambda = 0 \end{cases}$$

$$N_t^* = 1 - N_t^{-1} \qquad M_t^* = \frac{1 - M_t^{-1.5}}{1.5} \qquad W_t^* = \frac{1 - W_t^{-0.5}}{0.5}$$

Transformed HPI

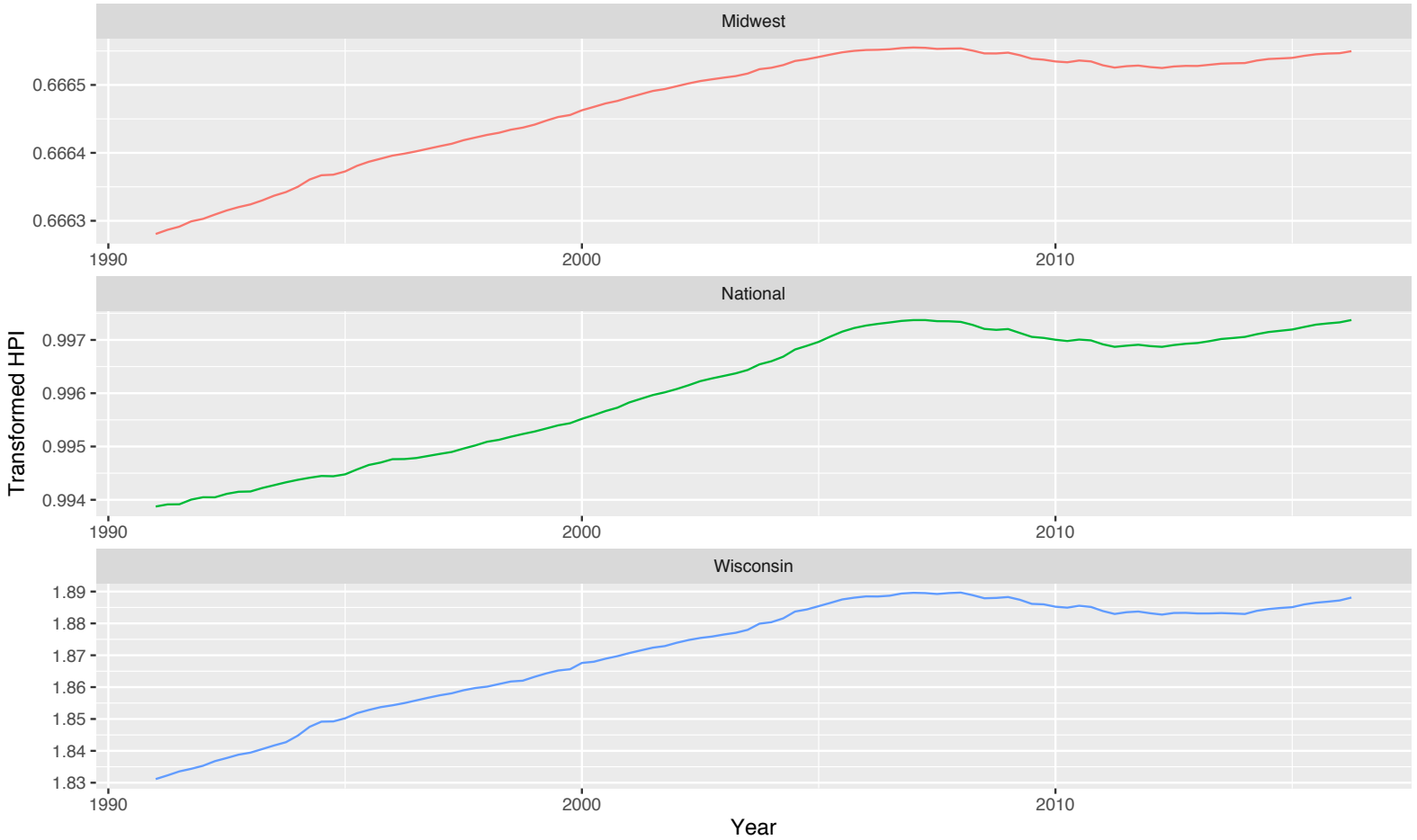


Figure: Quarterly HPI for each region after using the Box-Cox transformation.

Augmented Dickey-Fuller Test

Series	Y_t^*	∇Y_t^*	$\nabla^2 Y_t^*$
National	0.5237	0.6559	< 0.01
Midwest	0.4488	0.7507	< 0.01
Wisconsin	0.5581	0.6327	< 0.01

Table: P-values from the *Augmented Dickey-Fuller* for the transformed data, first difference, and second difference, respectively, for each series.

Differenced series'

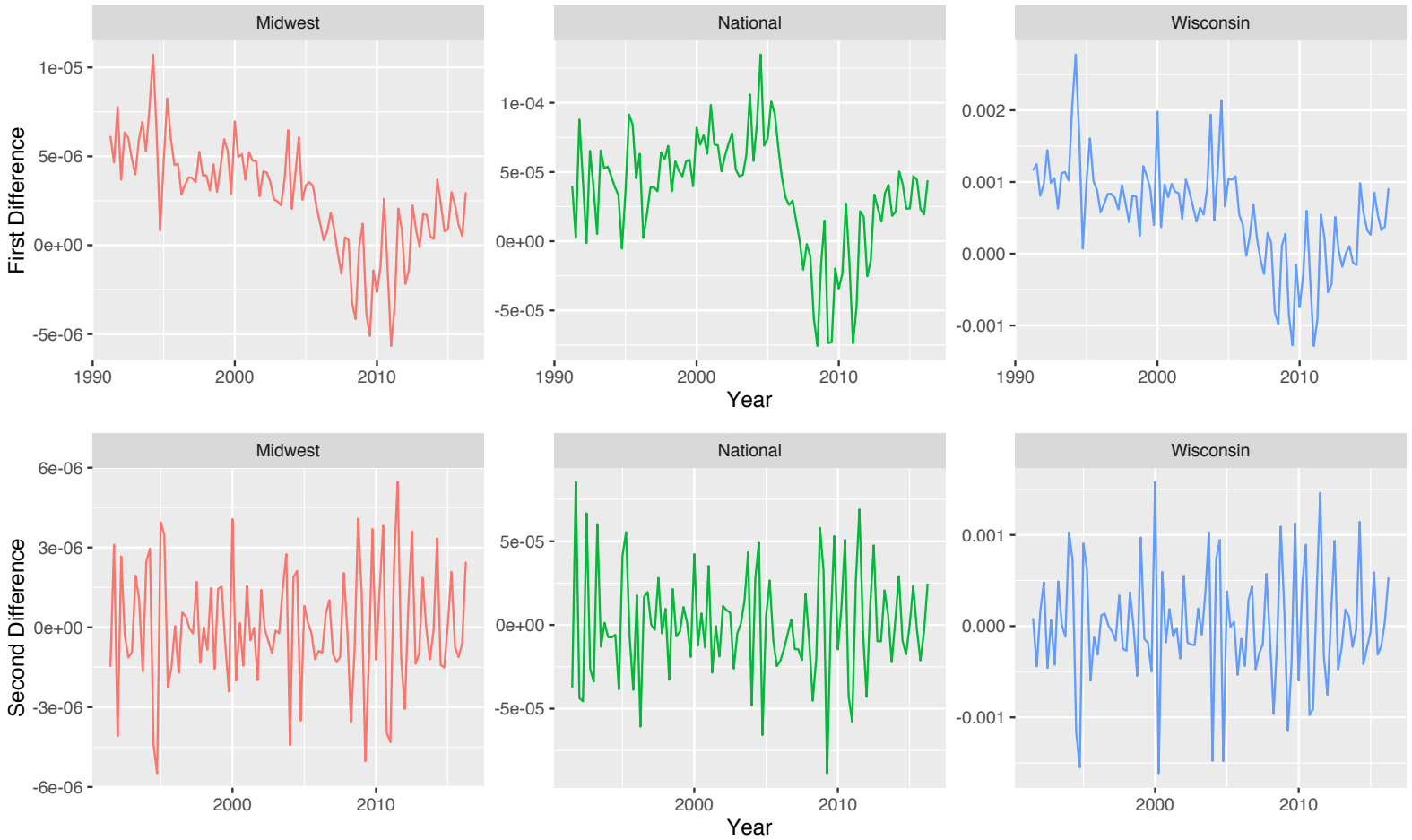


Figure: Plots for each of the three series after taking the *first difference* (top), and the *second difference* (bottom)

Autocorrelation of $\nabla^2 Y_t^*$

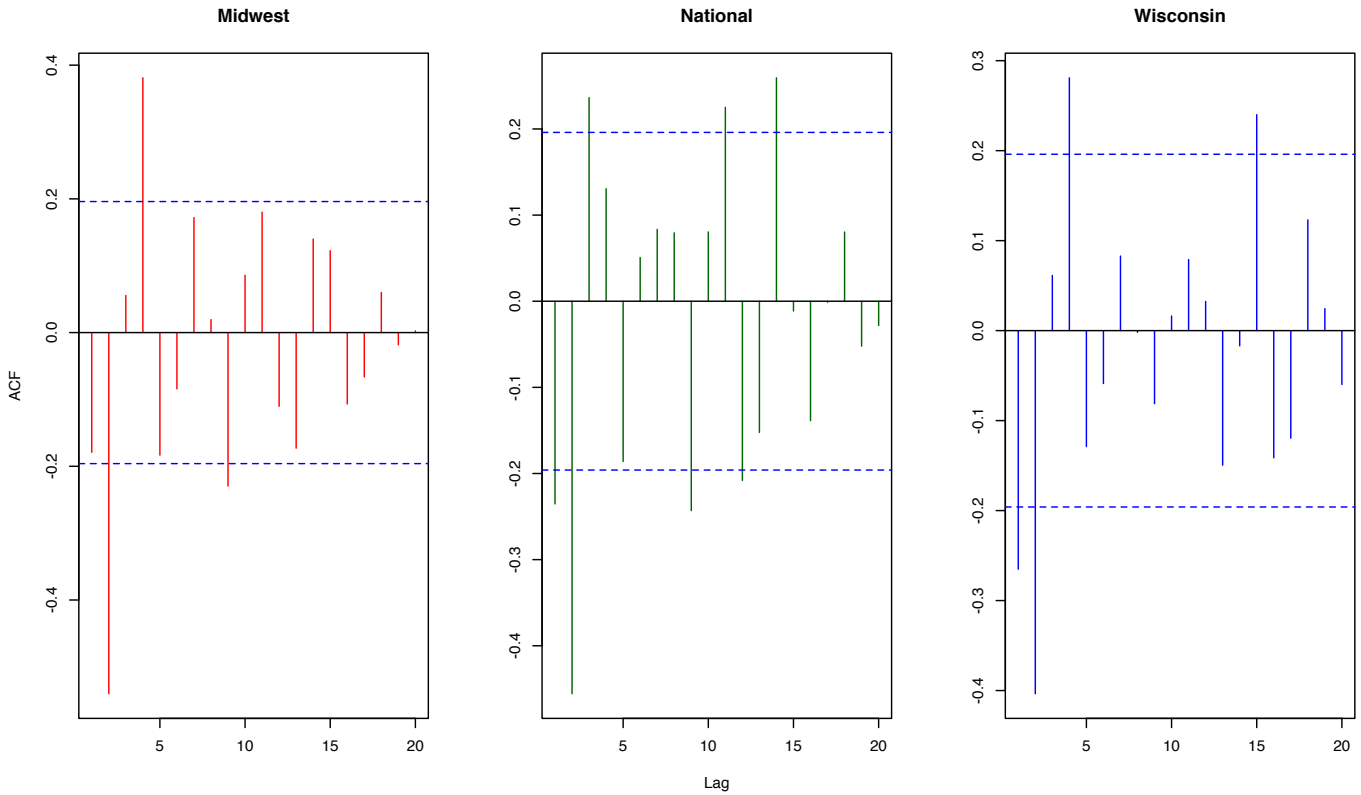


Figure: Autocorrelation for each (stationary) series after differencing twice.

Autocorrelation of $\nabla^2 Y_t^*$

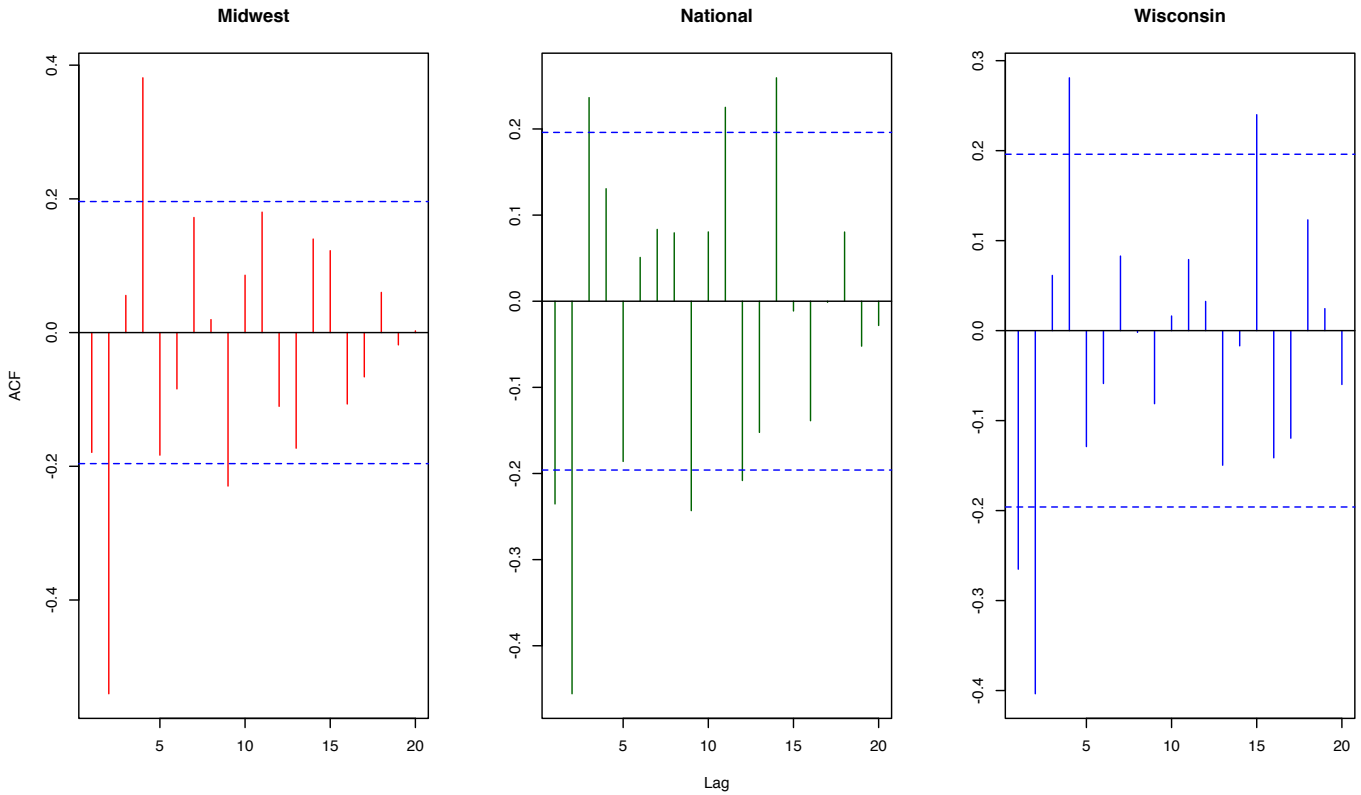


Figure: Autocorrelation for each (stationary) series after differencing twice.

$MA(2), MA(3), MA(4), AR(2)$

Partial-autocorrelation of $\nabla^2 Y_t^*$

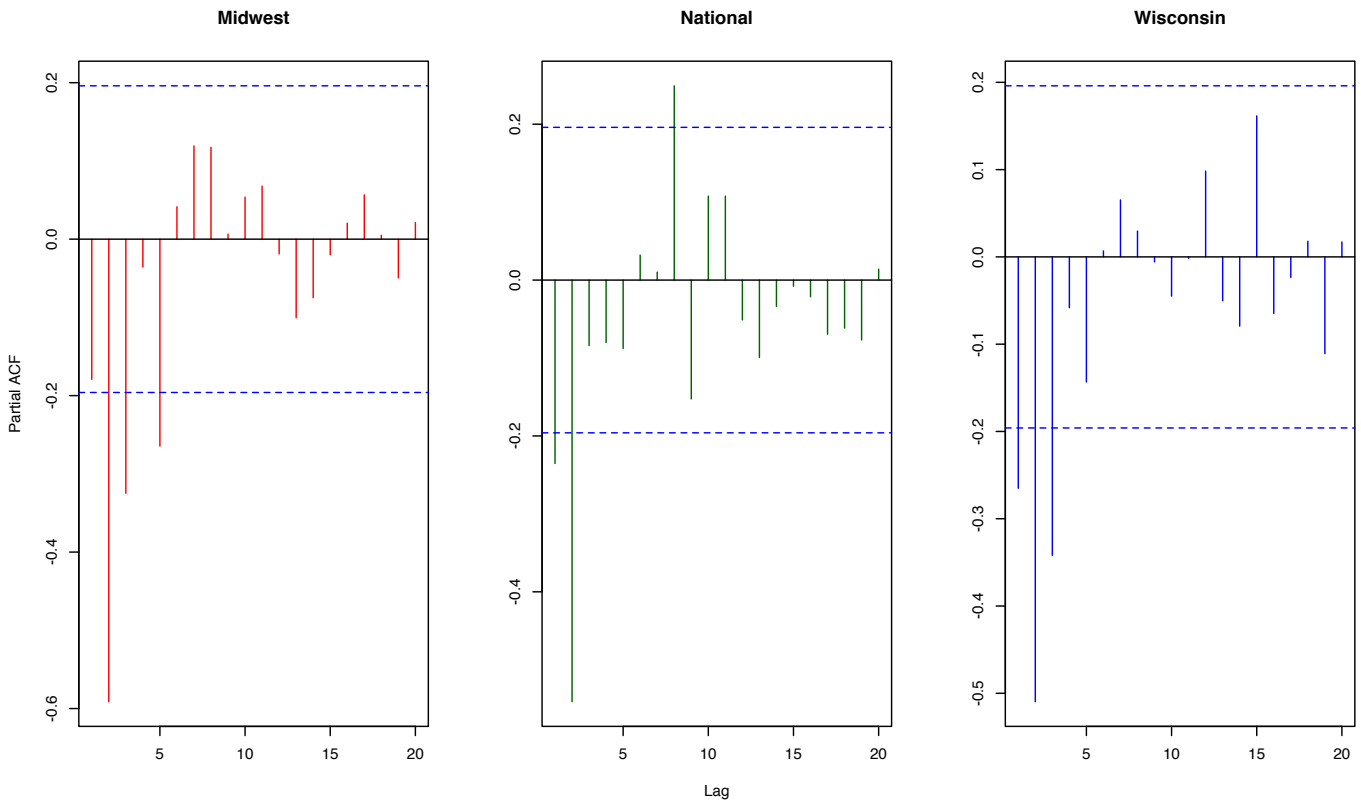


Figure: Partial-autocorrelation for each (stationary) series after differencing twice.

Partial-autocorrelation of $\nabla^2 Y_t^*$

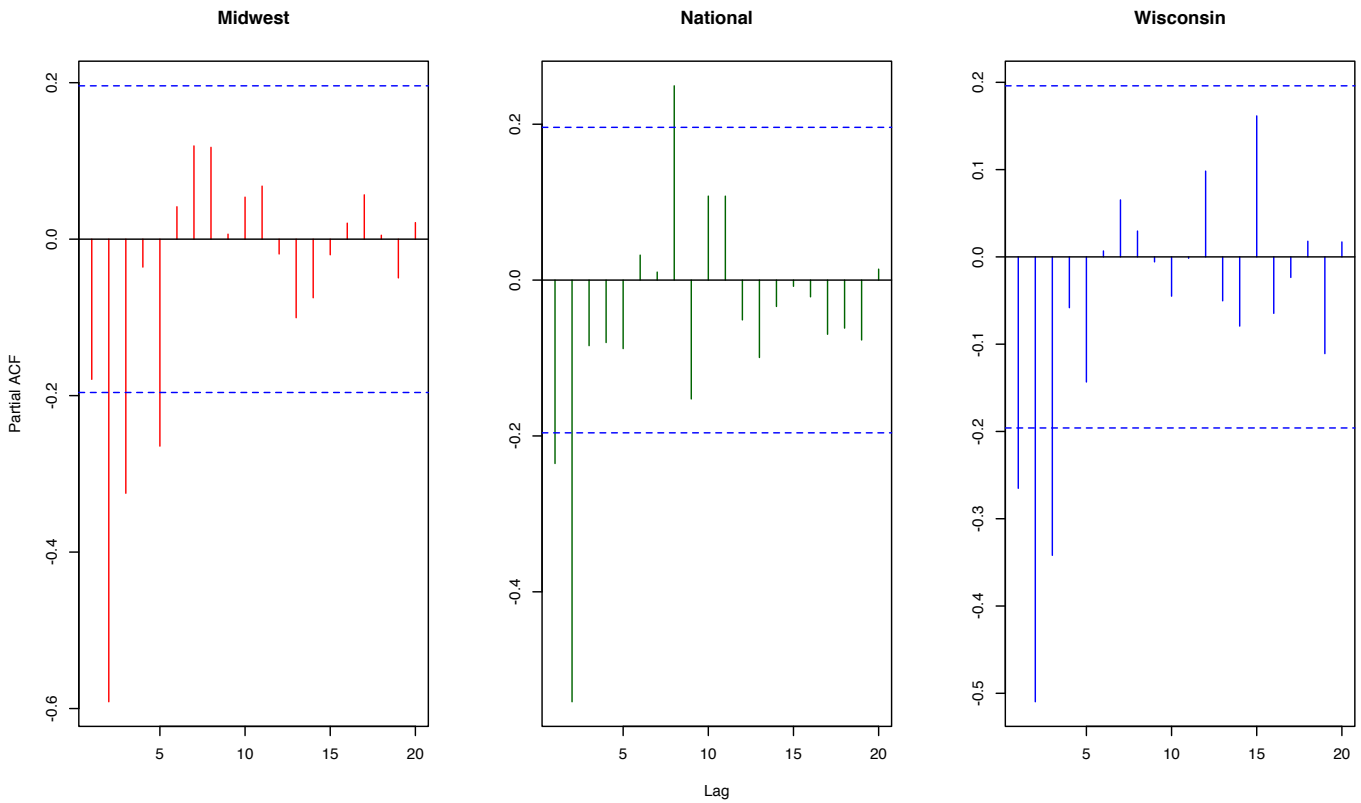


Figure: Partial-autocorrelation for each (stationary) series after differencing twice.

$AR(2), AR(3), AR(5)$

armasubsets

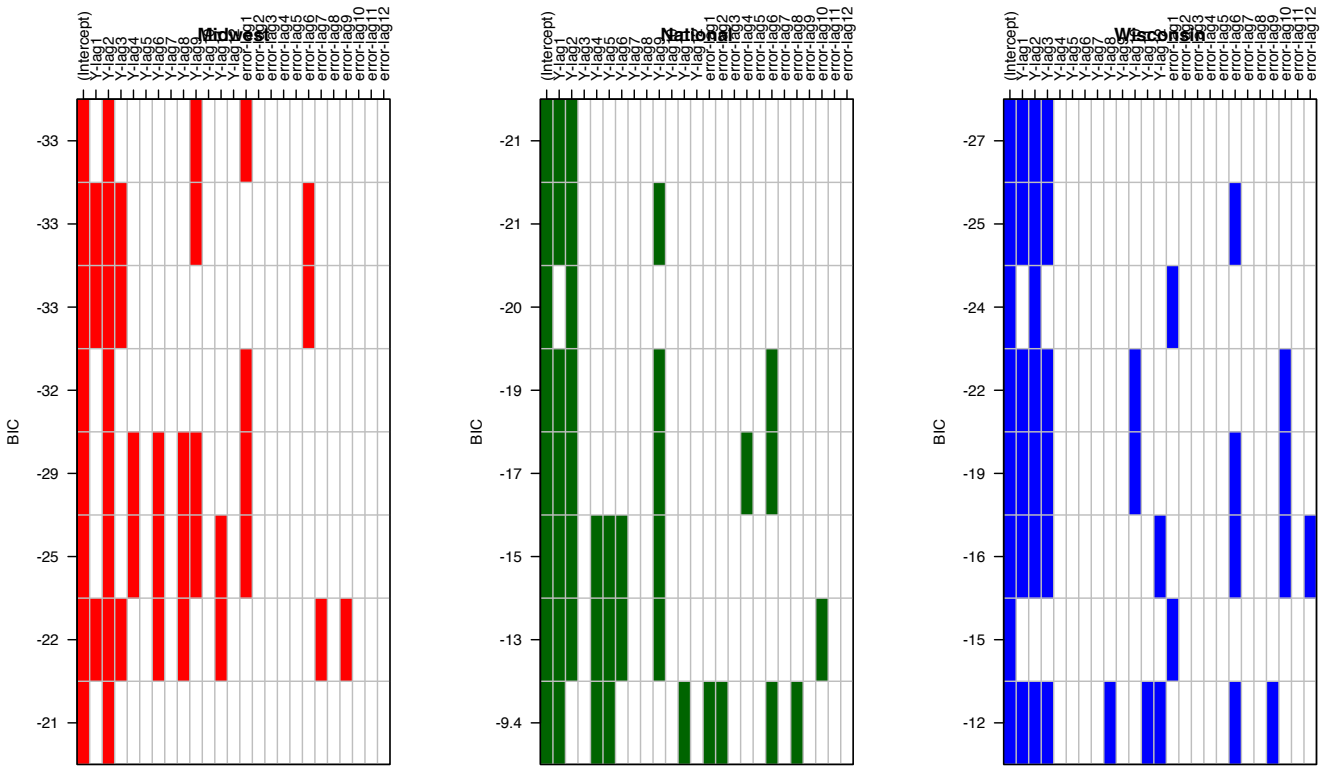
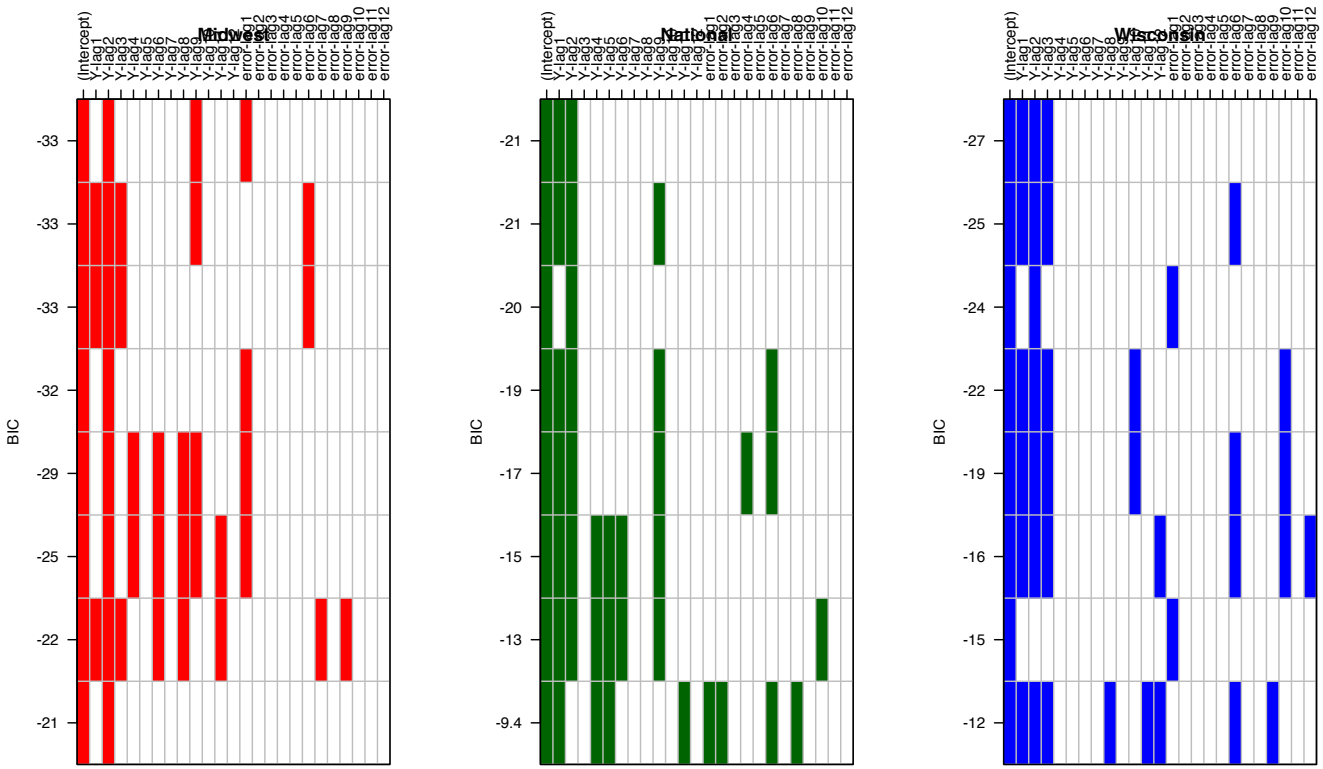


Figure: Plots produced by `armasubsets` suggesting potential models for each series according to the BIC criterion.

armasubsets



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$ARMA(9,1), AR(2), AR(3)$

Final Model Pool

$$AR(2) \quad AR(3) \quad AR(5) \quad MA(3) \quad MA(4)$$

$$ARMA(2,3) \quad ARMA(2,6) \quad ARMA(9,1)$$

Reducing Pool with AICc

Series	AR(2)	AR(3)	AR(5)	MA(3)	MA(4)
National	**-1826.012**	-1824.055	-1821.335	-1820.946	-1822.218
Midwest	-2362.055	-2370.101	**-2373.637**	-2363.714	-2372.163
Wisconsin	-1221.921	**-1232.339**	-1230.323	-1228.139	-1228.611

Series	ARMA(2,3)	ARMA(2,6)	ARMA(9,1)
National	**-1826.98**	-1824.153	-1824.934
Midwest	-2367.418	-2368.114	**-2373.077**
Wisconsin	-1228.54	-1227.224	**-1232.435**

Table: Corrected AIC values for potential models of each series. Bolded values are within two of minimum of the corresponding row.

Choosing Final Model with MAD

$$MAD = \frac{\sum_{i=1}^{n} |\nabla^2 Y_t^* - \widehat{\nabla^2 Y_t^*}|}{n}$$

where $\widehat{\nabla^2 Y_t^*}$ is the predicted value of the second difference.

National	MAD	Midwest	MAD	Wisconsin	MAD
AR(2)	0.125	AR(5)	0.00994	AR(3)	2.072
ARMA(2,3)	0.142	ARMA(9,1)	0.00987	ARMA(9,1)	4.080

Table: Mean absolute difference of the last 5 observations ($\times$ 10000) for the best two models according to AICc for each series.

Residual Diagnostics

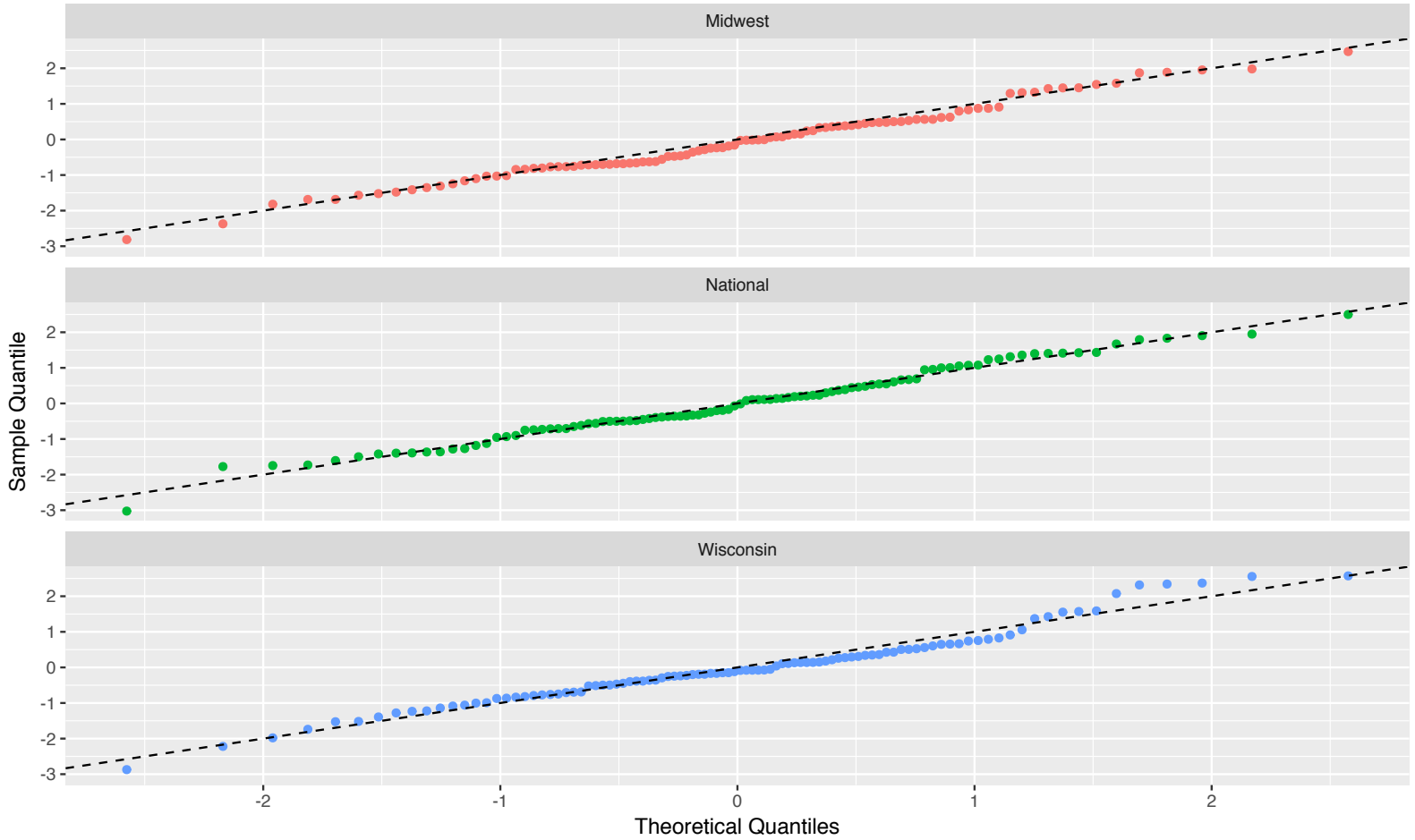


Figure: Quantile-quantile plots of the standardized residuals for the final model chosen on each series.

Residual Diagnostics

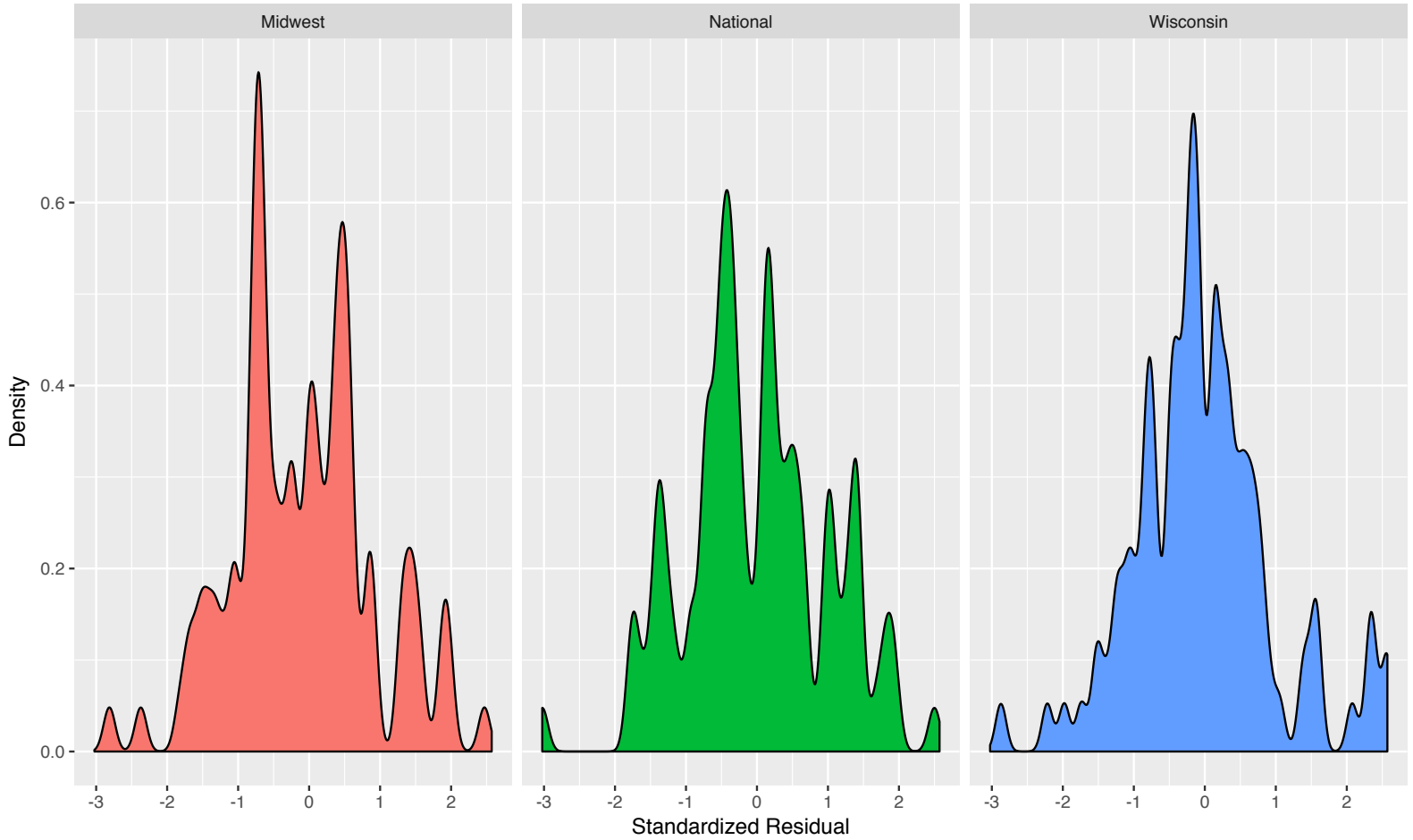


Figure: Density plots of the standardized residuals for the final model chosen on each series. A flexible bandwidth was chosen to ensure an accurate check for normality.

Residual Diagnostics

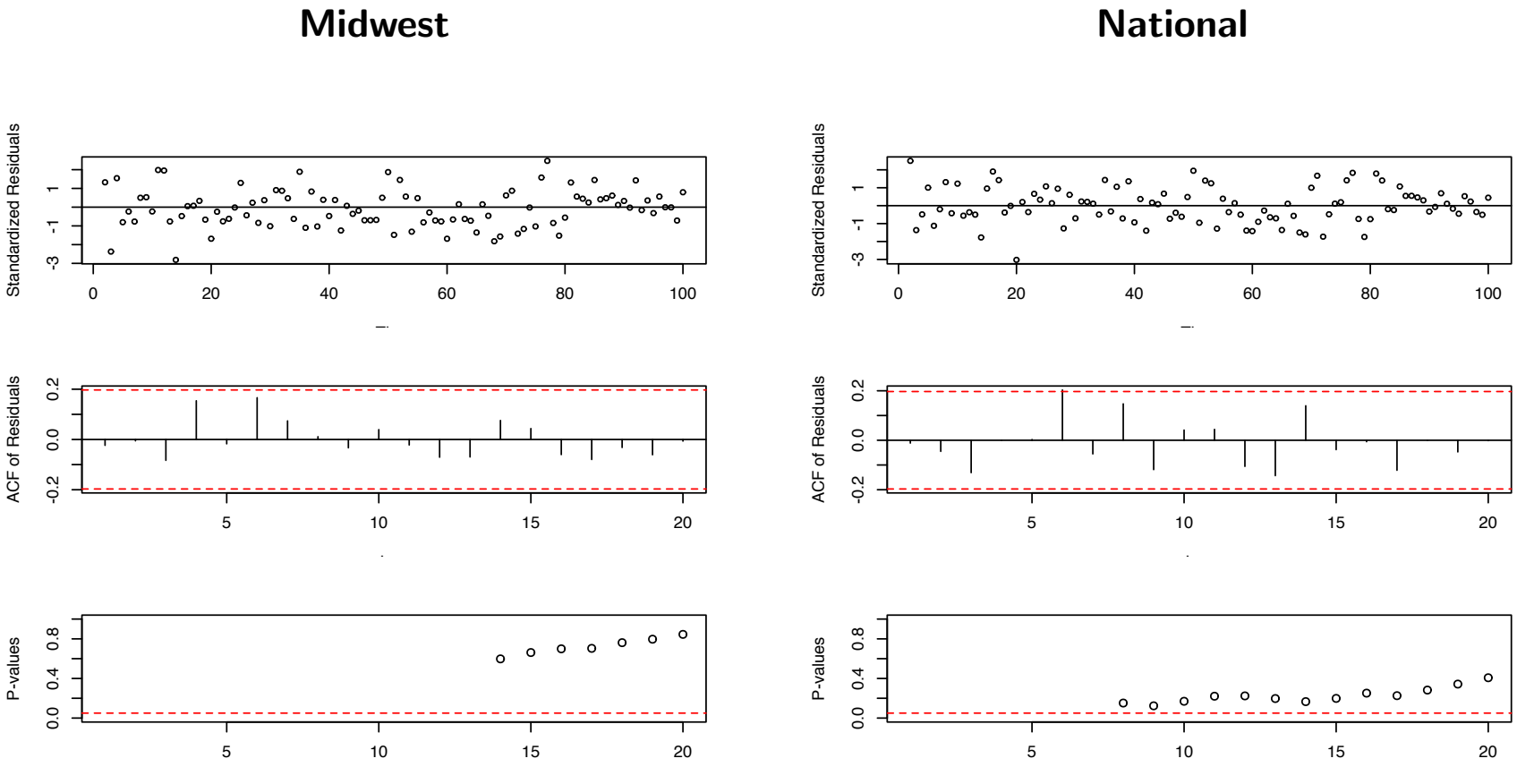
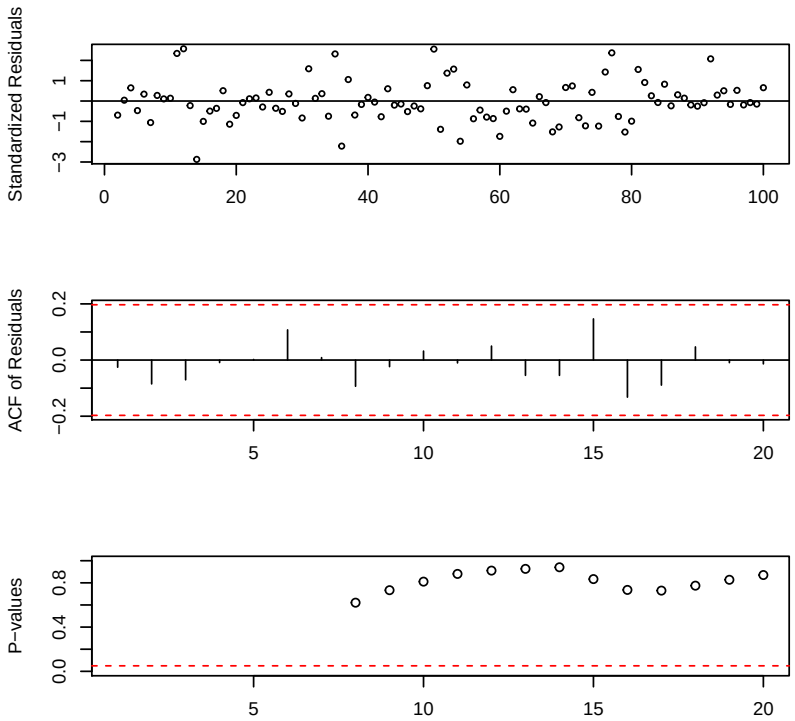


Table: Figure 10: Model diagnostic plots produced by `tsdiag` for each series containing a time plot and autocorrelation plot for the standardized residuals, as well as the Ljung-Box test for a number of lags.

Residual Diagnostics

Wisconsin



Residual Diagnostics

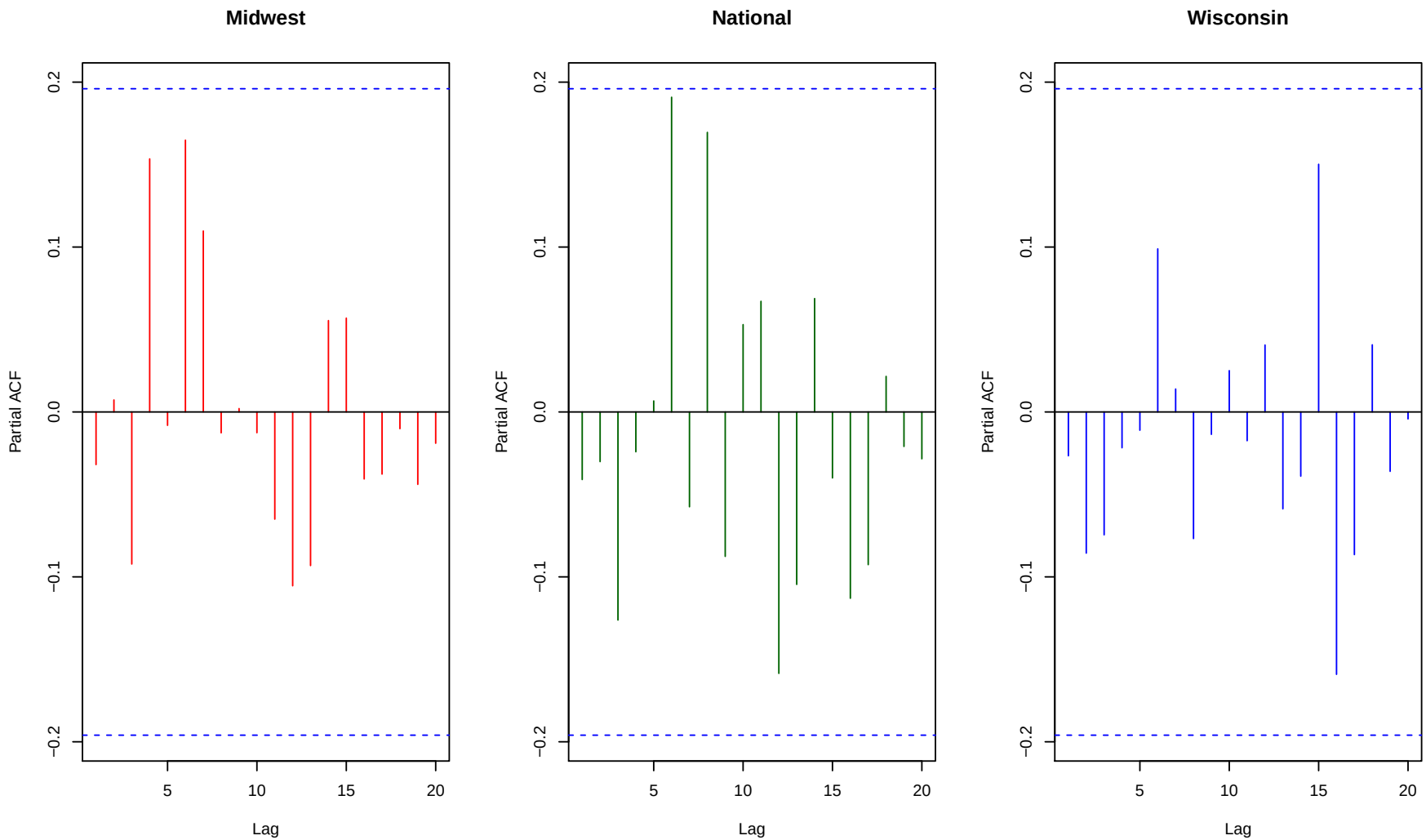


Figure: *

Forecasting

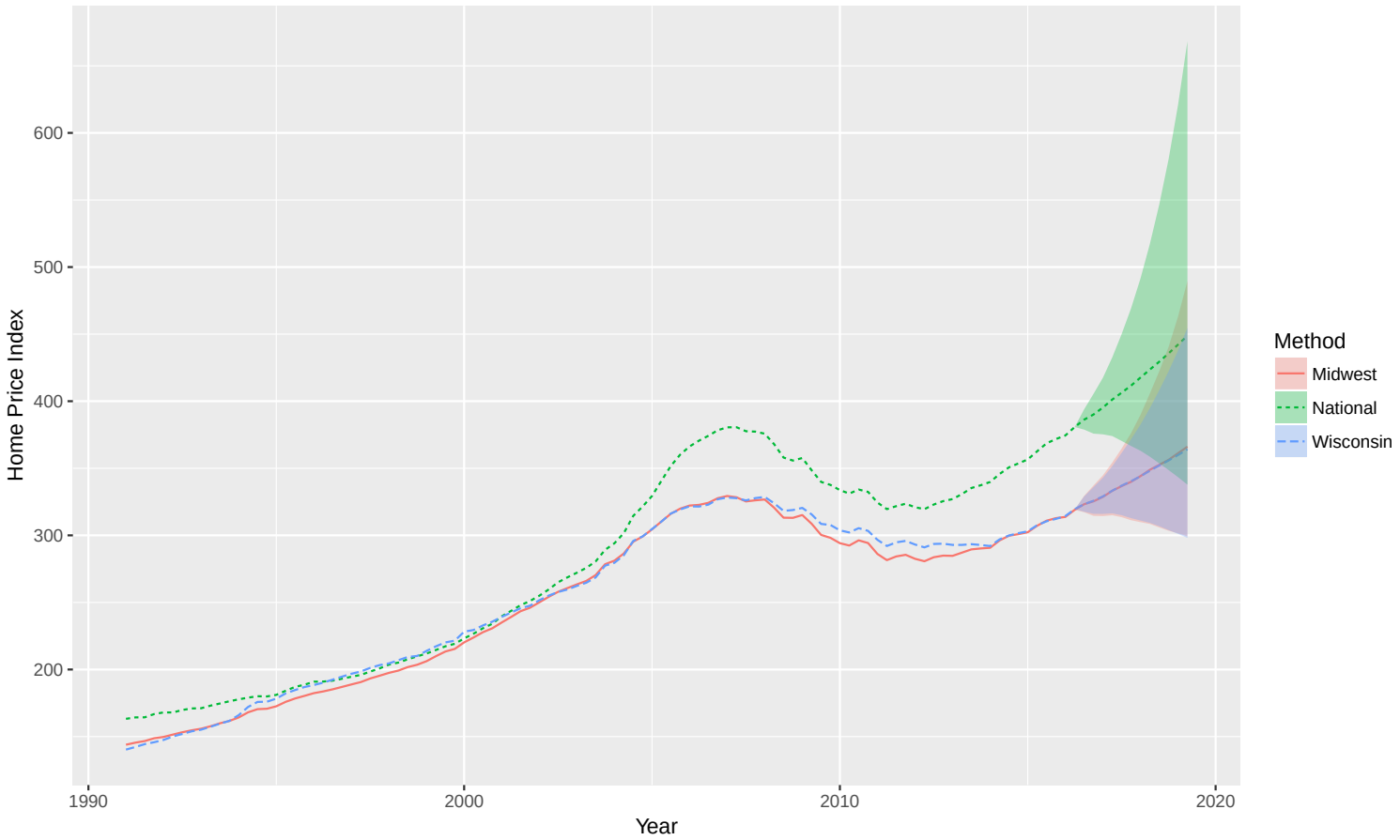


Figure: HPI forecasts with 95% prediction intervals for 2016-Q2 through 2019-Q2.

Forecasting

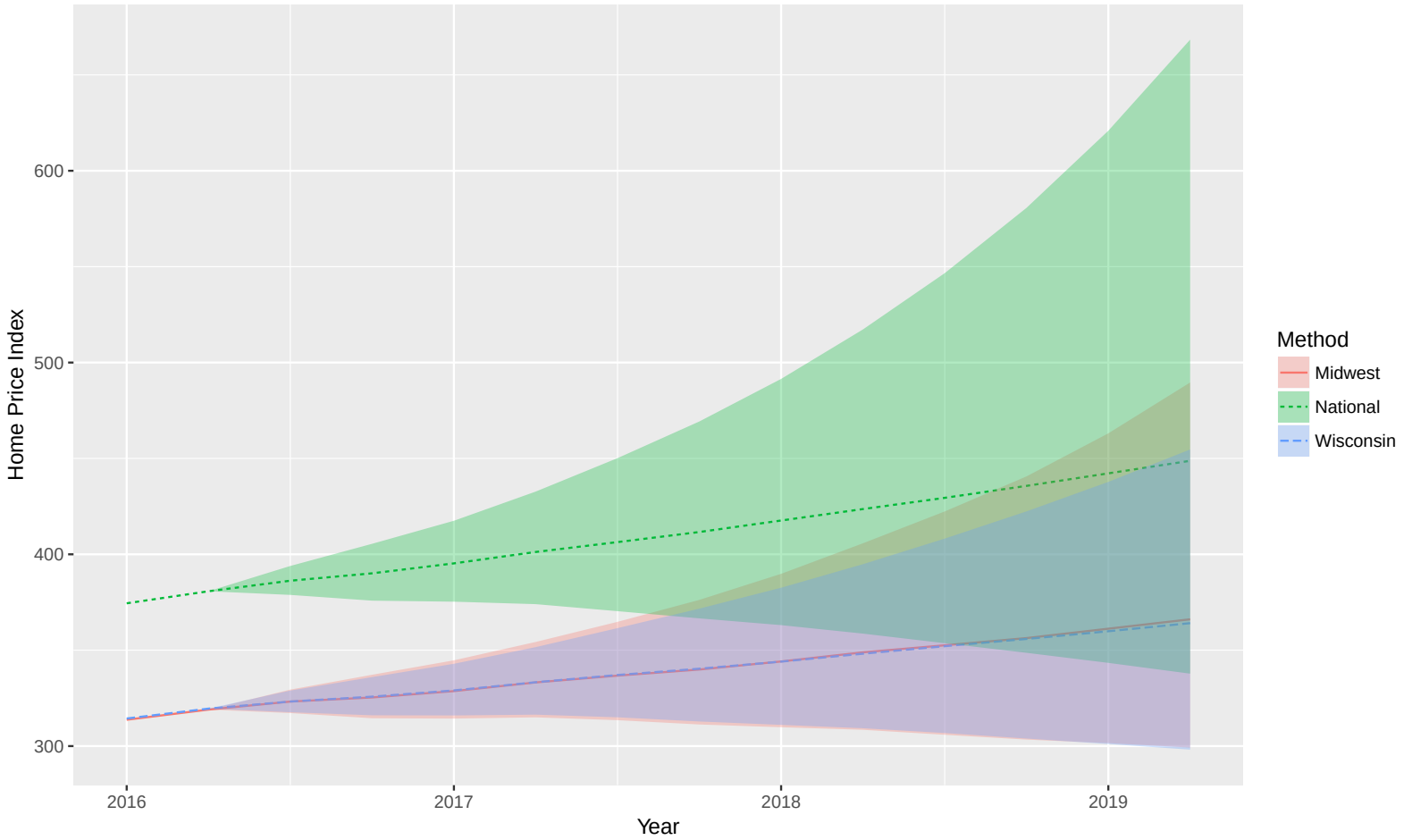


Figure: *

Figure 13: Zoomed plot of the 3-year forecast to assess predictions.

Forecasts

Year	Quarter	Midwest	National	Wisconsin
2016	3	323.20	386.23*	323.23
	4	325.36	390.09*	325.78
2017	1	328.66	395.28*	329.03
	2	333.20	401.23*	333.30
	3	336.73	406.37*	337.07
	4	339.92	411.63	340.35
2018	1	344.13	417.63	344.05
	2	348.88	423.59	348.16
	3	352.56	429.44	352.09
	4	356.32	435.69	355.87
2019	1	361.21	442.21	359.89
	2	366.10	448.76	364.11

Table: HPI forecasts for the following 3 years for each region. The asterisk(*) indicates non-overlapping 95% prediction intervals for the National series with both the Midwest and Wisconsin series'. Note that all estimates for the latter two are very similar, with almost identical prediction intervals.